

# Subvacuum effects in Quantum Critical Theories from Holographic Approach

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## Abstract

The subvacuum phenomena, induced by the squeezed vacuum of the strongly coupled quantum critical fields with a dynamical scaling  $z$ , are explored by a probe particle. The holographic description corresponds to a string moving in 4+1-dimensional Lifshitz geometry with gravitational wave perturbations. The dynamics of the particle can be realized from the motion of the endpoint of the string at the boundary. We then examine the particle's velocity dispersion, influenced by the squeezed vacuum states of the strongly coupled quantum critical fields. With appropriate choices of squeezing parameters, the velocity dispersion is found smaller than the value caused by the normal vacuum fluctuations of the fields. This leads to the subvacuum effect. We find that the large coupling constant of the quantum fields tends to counteract the effect in reduction of velocity dispersion, though this phenomenon is in principle observable. The effect of the squeezed vacuum on the decoherence dynamics of a quantum particle is also investigated. Coherence loss can be shown less severe in certain squeezed vacuums than in normal vacuum. This recovery of coherence is understood as recoherence, another manifestation of the subvacuum phenomena. We make some estimates of the degree of recoherence and find that, on contrary to the velocity dispersion case, the recoherence effect is enhanced by the large coupling constant. Finally we compare the findings in our earlier works when the particle is influenced by a weakly coupled relativistic field with the dynamical scaling  $z = 1$ .

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## I. INTRODUCTION

Engineering vacuum state may render suppression of its quantum fluctuations, resulting in the so-called subvacuum phenomenon. The existence of negative energy density of the quantum field is a renowned example, where the renormalized expectation value of the energy density operator can become negative in some spacetime regions [1]. However if the dynamics of quantum field theory places no restrictions on negative energy density, it may produce significant macroscopic effects that potentially violate the second law of thermodynamics [2, 3] or the cosmic censorship [4]. The negative energy density may also imply exotic phenomena such as traversable wormholes [5] and warp drive [6]. Thus, it has been shown that the renormalized local energy density can not be arbitrarily negative for an arbitrarily long period of time. There exists an inequality, constraining the magnitude and duration of the negative energy density [4, 7–9]. Different aspects of the subvacuum effects of quantum field theory can be realized by considering these effects on the dynamics of a particle, with which the field couples. It is found that the velocity dispersion of the particle, induced by the squeezed vacuum of the quantum fields with appropriate choices of squeezing parameters, can be smaller than the value determined by the normal vacuum [11]. A similar subvacuum phenomenon has been investigated in the context of quantum decoherence of the particle state as a result of an unavoidable interaction with environmental quantum fields [12, 13]. This decoherence effect can be observed by the contrast change of the fringes in the interference pattern. It has been shown that coherence loss can be less severe in certain squeezed vacuums than in normal vacuum. This is known as recoherence. However, all of the above-mentioned subvacuum phenomena are explored in either free or weakly coupled field theories. Accordingly, their effects are supposedly rather weak. This prompts us to pursue these subvacuum effects arising from strongly coupled fields.

The idea of holographic duality was originally proposed as the correspondence between 4-dimensional conformal field theory (CFT) and gravity theory in 5-dimensional anti-de Sitter (AdS) space [14]. It is soon generalized to other backgrounds and field theories and opens up the possibility to study the strong coupling problems in the condensed matter systems and the hydrodynamics of the quark-gluon plasma (see [15, 16] for reviews). There have been considerable efforts of employing the holographic duality to explore the dissipative dynamics of a particle moving in a strongly coupled environment [17–19]. In these cases, the endpoint

of the string on the boundary of the AdS black hole serves as a probed particle. Thereafter more works were devoted to understanding the fluctuations of this endpoint of the string in terms of Brownian motion in general backgrounds [20–46]. A review on the holographic Brownian motion can be found in [47].

The holographic duality provides a phenomenological description of strongly coupled physics, but its applicability for a real physical system has to be justified by its success in explaining and predicting experimental results. Thus, our current work will explore the subvacuum effects on a particle in squeezed vacuum of strongly coupled quantum critical fields by the method of holographic influence functional. This method was developed by us [48], and is consistent with the one in [49–51]. It is hoped that the large coupling constant might offer the possibility to observe these effects. The dual description for a particle coupled to quantum critical fields in its normal vacuum state corresponds to a string hanging from the boundary of  $4+1$ -dimensional Lifshitz geometry. The endpoint of a string is identified as the particle’s position. Thus, the effects of the quantum critical fields on the dynamics of the particle will be encoded in the Green’s functions of the associated boundary fields. Additionally, a possible holographically realization of the squeezing vacuum of the boundary fields is given by the gravitation wave perturbations in Lifshitz geometry [56]. The corrections to the Green’s functions of the boundary fields can be found from the perturbed holographic influence functional in bulk geometry, altered by the gravitational wave. In the context of holographic Brownian motion, the squeezed-vacuum correlation functions of the string’s endpoint can also be constructed through the Bogoliubov transformations from the normal vacuum state. The corresponding Green’s functions of boundary fields can then be obtained via the Langevin equation, derived from holographic influence functional. Comparing these two results, we find the forms of the Hadamard functions of boundary field, in leading order of the small squeezing parameter and gravitational wave perturbations, allow the identification of the squeezing parameter with the boundary value of the gravitational wave perturbation. The Bogoliubov transformations leave the retarded Green’s function unchanged. This can be confirmed by looking at the holographic retarded Green’s function to leading order in gravitational wave perturbation. Then we are able to explore the effects from squeezed vacuum of the strongly coupled fields on the dynamics of a particle.

Our presentation is organized as follows. In next section, we briefly review the method of holographic influence functional. The environmental degrees of freedom in the full density

matrix are traced over to obtain the reduced density matrix of the system. Their effects are all encoded in the influence functional. We then construct the holographic influence functional for a probed string in Lifshitz geometry. Later the Lifshitz geometry is perturbed by gravitational waves. The perturbed holographic influence functional is found, from which the nonequilibrium Green's functions of boundary fields are obtained from this bulk construction. In Sec. III, the correlation functions of the string's end point in its squeezed vacuum states can also be constructed via the Bogoliubov transformations from the normal vacuum state of the string. With the derived Langevin equation of the string's end point, it allows to identify the possible holographic dual of the squeezed vacuum states as gravitation waves perturbations. We then study subvacuum effects on particle's velocity dispersion influenced by squeezed vacuum fluctuations. In Sec. IV, the decoherence dynamics of a particle affected by the squeezed vacuum of quantum critical fields are also explored. The reduction in quantum coherence is measured by the decoherence functional, given by the holographic influence functional. We propose an interference experiment to find the subvacuum effect of recoherence. Concluding remarks are in Sec. V.

## II. HOLOGRAPHIC INFLUENCE FUNCTIONAL AND CORRELATORS FOR THE SQUEEZE VACUUM STATES

### A. Influence functional in field theory

We first review the method of influence functional in field theory. When the system of interest couples with the environment, their full dynamics can be described by the density matrix  $\rho(t)$  that evolves unitarily according to

$$\rho(t_f) = U(t_f, t_i) \rho(t_i) U^{-1}(t_f, t_i) \quad (1)$$

with  $U(t_f, t_i)$  the time evolution operator of the system and environment. The reduced density matrix  $\rho_r$  is obtained by tracing over the environmental degrees of freedom in the full density matrix, and it will include all the effects from the environment on the system. We assume that the initial density matrix at time  $t_i$  is factorized as

$$\rho(t_i) = \rho_q(t_i) \otimes \rho_F(t_i), \quad (2)$$

for simplicity, where  $q$  and  $F$  generically represent the system and the environment variables respectively. We further assume that the environment field is in thermal equilibrium at temperature  $T = 1/\beta$  before it is brought into contact with the system, so  $\rho_F(t_i)$  takes the form

$$\rho_F(t_i) = e^{-\beta H_F}, \quad (3)$$

where  $H_F$  is the Hamiltonian for the  $F$  field. The vacuum state can be obtained by taking the zero- $T$  limit. We consider the system linearly coupled to an environment field. The full Lagrangian takes this form

$$L(q, F) = L_q[q] + L_F[F] + qF. \quad (4)$$

Then the reduced density matrix becomes [53–55]

$$\rho_r(q_f, \tilde{q}_f, t_f) = \int dq_1 dq_2 \mathcal{J}(q_f, \tilde{q}_f, t_f; q_1, q_2, t_i) \rho_q(q_1, q_2, t_i), \quad (5)$$

where the propagating function  $\mathcal{J}(q_f, \tilde{q}_f, t_f; q_1, q_2, t_i)$  is

$$\mathcal{J}(q_f, \tilde{q}_f, t_f; q_1, q_2, t_i) = \int_{q_1}^{q_f} \mathcal{D}q^+ \int_{q_2}^{\tilde{q}_f} \mathcal{D}q^- \exp \left[ i \int_{t_i}^{t_f} dt (L_q[q^+] - L_q[q^-]) \right] \mathcal{F}[q^+, q^-]. \quad (6)$$

The influence functional  $\mathcal{F}[q^+, q^-]$  can be written in terms of the real-time Green's functions [57],

$$\begin{aligned} \mathcal{F}[q^+, q^-] &= \exp \left\{ -\frac{i}{2} \int_{t_i}^{t_f} dt \int_{t_i}^{t_f} dt' \left[ q^+(t) G^{++}(t, t') q^+(t') - q^+(t) G^{+-}(t, t') q^-(t') \right. \right. \\ &\quad \left. \left. - q^-(t) G^{-+}(t, t') q^+(t') + q^-(t) G^{--}(t, t') q^-(t') \right] \right\}, \end{aligned} \quad (7)$$

where we keep the terms to the quadratic order in the particle position, valid in the linear response approximation. The Green's functions involved are time-ordered, anti-time-ordered and Wightman functions, defined as

$$\begin{aligned} i G^{+-}(t, t') &= \langle F(t') F(t) \rangle, \\ i G^{-+}(t, t') &= \langle F(t) F(t') \rangle, \\ i G^{++}(t, t') &= \langle F(t) F(t') \rangle \theta(t - t') + \langle F(t') F(t) \rangle \theta(t' - t), \\ i G^{--}(t, t') &= \langle F(t') F(t) \rangle \theta(t - t') + \langle F(t) F(t') \rangle \theta(t' - t). \end{aligned} \quad (8)$$

The retarded Green's function and Hadamard function can be constructed from them according to

$$\begin{aligned} G_R(t-t') &\equiv -i\theta(t-t')\langle[F(t), F(t')]\rangle = \left\{G^{++}(t, t') - G^{+-}(t, t')\right\}, \\ G_H(t-t') &\equiv \frac{1}{2}\langle\{F(t), F(t')\}\rangle = \frac{i}{4}\left\{G^{++}(t, t') + G^{+-}(t, t') + G^{--}(t, t') + G^{-+}(t, t')\right\}. \end{aligned} \quad (9)$$

In a time-translation invariant environment, the Fourier transform of various Green's functions is defined by

$$G(t-t') = \int \frac{d\omega}{2\pi} G(\omega) e^{-i\omega(t-t')}. \quad (10)$$

Their respective  $\omega$ -dependent functions are obtained as

$$\begin{aligned} G^{++}(\omega) &= \text{Re}G_R(\omega) + (1+2n)i \text{Im}G_R(\omega), \\ G^{--}(\omega) &= -\text{Re}G_R(\omega) + (1+2n)i \text{Im}G_R(\omega), \\ G^{+-}(\omega) &= 2ni \text{Im}G_R(\omega), \\ G^{-+}(\omega) &= 2(1+n)i \text{Im}G_R(\omega) \end{aligned} \quad (11)$$

with  $n = (e^{\frac{\omega}{T}} - 1)^{-1}$ . Notice that the above Green's functions are not totally independent and they obey the following relations:

$$G^{++}(\omega) + G^{--}(\omega) - G^{+-}(\omega) - G^{-+}(\omega) = 0, \quad (12)$$

resulting from the unitary evolution of the system and environment, and

$$\frac{G^{+-}(\omega)}{G^{-+}(\omega)} = e^{-\frac{\omega}{T}} \quad (13)$$

due to a bosonic thermal bath. Additionally the fluctuation-dissipation relation gives

$$G_H(\omega) = -(1+2n) \text{Im}G_R(\omega). \quad (14)$$

## B. Holographic influence functional

In this section we review the construction of the holographic influence functional from dual gravity theory. The conventional approach to derive the influence functional can at best be perturbatively implemented for a weakly coupled environment, let alone the strong coupled theory. Thus for the latter case, the holography method will be employed to find the

influence functional of the strongly coupled environment. We consider a particle coupled to quantum critical theories in 3+1-dimension at zero temperature [33, 40]. Its dual description is a straight string moving in the 4+1-dimensional Lifshitz geometry with the metric

$$ds^2 = -\frac{r^{2z}}{L^{2z}}dt^2 + \frac{L^2}{r^2}dr^2 + \frac{r^2}{L^2}d\vec{x}^2, \quad (15)$$

in which  $L$  is the radius of curvature of the geometry. This gravity background (15) can be engineered by coupling the gravitation field with negative cosmological constant to a massive vector field. The relevant action to give the above metric is [58],

$$S = \frac{1}{16\pi G_{4+1}} \int d^{4+1}x \sqrt{-g} (R + 2\Lambda - \frac{1}{4}\mathcal{F}^{\mu\nu}\mathcal{F}_{\mu\nu} - \frac{1}{2}m_A^2\mathcal{A}^\mu\mathcal{A}_\mu). \quad (16)$$

In addition to the Einstein-Hilbert action and the cosmological constant  $\Lambda$  term, the action of a vector field  $\mathcal{A}_\mu$  with mass  $m_A$  is introduced where  $\mathcal{F}_{\mu\nu}$  is the field strength of  $\mathcal{A}_\mu$ . The equations of motion for the metric and the vector field can be derived from this action, and they are

$$R_{\mu\nu} = -\frac{2}{3}\Lambda g_{\mu\nu} + \frac{1}{2}g^{\alpha\beta}\mathcal{F}_{\mu\alpha}\mathcal{F}_{\nu\beta} + \frac{1}{2}m_A^2\mathcal{A}_\mu\mathcal{A}_\nu - \frac{1}{12}\mathcal{F}_{\alpha\beta}\mathcal{F}^{\beta\alpha}g_{\mu\nu}, \quad (17)$$

$$D_\mu\mathcal{F}^{\mu\nu} = m_A^2\mathcal{A}^\nu, \quad (18)$$

where  $D_\mu$  is a covariant derivative with respect to the background metric  $g_{\mu\nu}$ . The solutions of the vector field are assumed to be

$$\mathcal{A}_\mu^{(0)} = \mathcal{A} \frac{r^z}{L^z} \delta_\mu^0. \quad (19)$$

Then the Lifshitz background with metric (15) can be achieved by setting

$$\mathcal{A} = \sqrt{\frac{2(z-1)}{z}}, \quad m_A^2 = \frac{3z}{L^2}, \quad \Lambda = \frac{9+2z-z^2}{2L^2}. \quad (20)$$

Using the method of holography, as in [33, 40], the classical on-shell gravity action of the string moving in the Lifshitz background can be identified as the influence functional for a particle in strongly coupled field theory. We start from considering the finite-temperature nonequilibrium correlators and the zero temperature limit will be taken later. Hereafter,  $L = 1$  will be used, and reintroduced later when needed. The metric of Lifshitz black hole has the form

$$ds^2 = -r^{2z}f(r)dt^2 + \frac{dr^2}{f(r)r^2} + r^2d\vec{x}^2, \quad (21)$$

where  $f(r) \rightarrow 1$  for  $r \rightarrow \infty$  and  $f(r) \simeq c(r - r_h)$  near the black hole horizon  $r_h$  with  $c = (z + 3)/r_h$ . The detailed form of  $f(r)$  is not relevant since only the low-frequency perturbation is considered in our subsequent discussions. The black hole temperature, which is also the temperature in the boundary field theory, is

$$\frac{1}{T} = \frac{4\pi}{z + 3} \frac{1}{r_h^z}. \quad (22)$$

Here we assume that the string moves only along the  $x$ -direction and its position variable is  $X$ . It is then straightforward to obtain the linearized Nambu-Goto action for a string in the background of Lifshitz black hole:

$$S_{NG} = -\frac{1}{4\pi\alpha'} \int dr dt \left( r^{z+3} f(r) \partial_r X \partial_r X - \frac{\partial_t X \partial_t X}{f(r) r^{z-1}} \right), \quad (23)$$

where  $X(t, r)$  is the linearized string perturbation in the static gauge. The equation of motion of the string is

$$\frac{\partial}{\partial r} \left[ r^{z+3} f(r) \frac{\partial}{\partial r} X(r, t) \right] - \frac{\partial}{\partial t} \left[ \frac{1}{r^{z-1} f(r)} \frac{\partial}{\partial t} X(r, t) \right] = 0. \quad (24)$$

We express the solutions in terms of two linearly independent solutions, whose Fourier transformations are complex conjugates to one another,  $\mathcal{X}_\omega(r)$  and  $\mathcal{X}_\omega^*(r)$  and have the properties  $\mathcal{X}_\omega(r) \underset{r \rightarrow r_h}{\propto} e^{+i\omega r_*}$  and  $\mathcal{X}_\omega^*(r) \underset{r \rightarrow r_h}{\propto} e^{-i\omega r_*}$ . Here we have defined

$$r_* = \int dr \frac{1}{r^{z+1} f(r)}, \quad (25)$$

and the normalization is such that  $\mathcal{X}_\omega(r_b) = 1$ .

In accordance with the closed-time-path formalism [53–55] we have discussed in the previous section, we introduce  $Q^+(t, r_1)$  and  $Q^-(t, r_2)$ , which are the string worldsheets living in maximally extended black hole geometry with two asymptotical boundaries [49, 50]. Then, by choosing appropriate boundary conditions for the perturbations of the string  $Q^\pm(t, r)$  in this background geometry, the classical on-shell action of the string can be identified as the influence functional for a particle affected by the boundary fields [49]:

$$\mathcal{F}[q^+, q^-] = S_{gravity}(Q^+(t, r_b), Q^-(t, r_b)), \quad (26)$$

where the gravity action is the action  $S_{NG}$  in (23). After some algebraic reduction, the on-shell action contains only the boundary terms and it takes the form

$$S_{NG}^{\text{on-shell}} \simeq -\frac{r_b^{(z+3)}}{4\pi\alpha'} \int dt (Q^+(t, r_b) \partial_r Q^+(t, r_b) - Q^-(t, r_b) \partial_r Q^-(t, r_b)). \quad (27)$$



The parameter  $r_b$  is the location of the boundary and serves as a cutoff scale in the radial direction to render the action finite. The boundary conditions of the string perturbations are

$$q^\pm(t) = Q^\pm(t, r_b), \quad (28)$$

in which the variable  $q(t)$  can be identified as the position of the Brownian particle. Following [48], which is consistent with [49, 50], we find  $Q^\pm(\omega, r)$  given by

$$\begin{aligned} Q^+(\omega, r_1) &= \frac{1}{1 - e^{-\frac{\omega}{T}}} \left[ (q^-(\omega) - e^{-\frac{\omega}{T}} q^+(\omega)) \mathcal{X}_\omega(r_1) + (q^+(\omega) - q^-(\omega)) \mathcal{X}_\omega^*(r_1) \right], \\ Q^-(\omega, r_2) &= \frac{1}{1 - e^{-\frac{\omega}{T}}} \left[ (q^-(\omega) - e^{-\frac{\omega}{T}} q^+(\omega)) \mathcal{X}_\omega(r_1) + e^{-\frac{\omega}{T}} (q^+(\omega) - q^-(\omega)) \mathcal{X}_\omega^*(r_1) \right]. \end{aligned} \quad (29)$$

This general solution is then substituted into the classical on-shell action (27). Using (7) and (9), the retarded Green's function at finite temperature is obtained to be

$$G_R(\omega) = \frac{r_b^{z+3}}{2\pi\alpha'} \mathcal{X}_{-\omega}(r_b) \partial_r \mathcal{X}_\omega(r_b). \quad (30)$$

In general, the analytical expression of  $G_R(\omega)$  is not available except in the small  $\omega$  limit. Nevertheless, there is an analytical solution for  $\mathcal{X}_\omega(r)$  at zero temperature,

$$\mathcal{X}_\omega(r) = \frac{r_b^{1+\frac{z}{2}} H_{\frac{1}{z}+\frac{1}{2}}^{(1)}\left(\frac{\omega}{zr^{\frac{z}{2}}}\right)}{r^{1+\frac{z}{2}} H_{\frac{1}{z}+\frac{1}{2}}^{(1)}\left(\frac{\omega}{zr_b^{\frac{z}{2}}}\right)}. \quad (31)$$

Hence the zero-temperature retarded Green's function can be found for  $\omega > 0$  to be [33],

$$G_R^{(0)}(\omega) = -\frac{\omega r_b^2}{2\pi\alpha'} \frac{H_{\frac{1}{z}-\frac{1}{2}}^{(1)}\left(\frac{\omega}{zr_b^{\frac{z}{2}}}\right)}{H_{\frac{1}{z}+\frac{1}{2}}^{(1)}\left(\frac{\omega}{zr_b^{\frac{z}{2}}}\right)}. \quad (32)$$

All other correlators can be derived from (11) by taking the  $T \rightarrow 0$  limit. In particular, through the fluctuation-dissipation relation (14) in the  $T \rightarrow 0$  limit, we obtain the Hadamard function for  $\omega > 0$  as follows:

$$G_H^{(0)}(\omega) = \frac{zr_b^{2+z}}{\pi^2\alpha'} \frac{1}{J_{\frac{1}{z}+\frac{1}{2}}^2\left(\frac{\omega}{zr_b^{\frac{z}{2}}}\right) + Y_{\frac{1}{z}+\frac{1}{2}}^2\left(\frac{\omega}{zr_b^{\frac{z}{2}}}\right)}. \quad (33)$$

### C. Influence functional from gravitational wave perturbed geometry

We can engineer the vacuum of the boundary fields by perturbing the bulk geometry. In particular, as suggested in [56], we consider the gravitational wave perturbations in the

Lifshitz background with the metric  $g_{\mu\nu}^{(0)}$  (15). The perturbed metric is given by

$$g_{\mu\nu} = g_{\mu\nu}^{(0)} + r^2 \phi(t, r) \xi_{\mu\nu}, \quad (34)$$

where  $\xi_{\mu\nu}$ , the polarization tensor, has non-zero components only in the spatial directions of the boundary, and is assumed transverse and traceless. The field  $\phi(t, r)$  is introduced to parameterize small metric perturbations from gravitation waves, and its the equation of motion is,

$$r^{-2z} \partial_t^2 \phi(t, r) + (3 + z) r \partial_r \phi(t, r) + r^2 \partial_r^2 \phi(t, r) = 0 \quad (35)$$

which is obtained by linearizing (17) about the background solutions (15) and (19). Thus, the Fourier transform of the  $\phi(t, r)$  field in frequency space is defined as

$$\phi(t, r) = \int_0^\infty d\omega \phi(\omega, r) e^{-i\omega t} + \text{h.c.} \quad (36)$$

The normalizable solution of (35) can be given by

$$\phi(\omega, r) = r^{-\frac{2+z}{2}} \varphi(\omega) J_{\frac{2+z}{2z}} \left( \frac{\omega}{zr^z} \right). \quad (37)$$

The function  $\varphi(\omega)$  is determined by the boundary condition of the gravitation waves at  $r = r_b$ , and will be identified as the squeezing parameter defined below. Here we also assume that the string moves only along the  $x$ -direction with the position variable  $X$ . Then the Nambu-Goto action in perturbed Lifshitz geometry can be written explicitly as

$$S_{NG} = -\frac{1}{4\pi\alpha'} \int dr dt \left( r^{z+3} (1 + \phi(r, t)) \partial_r X \partial_r X - (1 + \phi(r, t)) \frac{\partial_t X \partial_t X}{r^{z-1}} \right). \quad (38)$$

Up to the first order in  $\phi$ , the equation of motion of the string becomes

$$\frac{\partial}{\partial r} \left[ r^{z+3} (1 + \phi(r, t)) \frac{\partial}{\partial r} X(r, t) \right] - \frac{\partial}{\partial t} \left[ \frac{1}{r^{z-1}} (1 + \phi(r, t)) \frac{\partial}{\partial t} X(r, t) \right] = 0. \quad (39)$$

We consider the perturbative solution which in frequency space is given by

$$X_\omega(r) = X_\omega^{(0)}(r) + X_\omega^{(\phi)}(r), \quad (40)$$

where the zeroth order solution (31) gives the retarded Green's function and Hadamard function in the unperturbed Lifshitz spacetime. Then the equation of motion for  $X_\omega^{(\phi)}(r)$  to leading order is given by

$$\begin{aligned} \frac{\partial}{\partial r} \left[ r^{z+3} \partial_r X_\omega^{(\phi)}(r) \right] + r^{1-z} \omega^2 X_\omega^{(\phi)}(r) &= - \int d\omega' \left[ r^{z+3} \partial_r \phi(\omega, r) \partial_r X_{\omega-\omega'}^{(0)}(r) \right. \\ &\quad \left. + \omega'(\omega - \omega') r^{1-z} \phi(\omega', r) X_{\omega-\omega'}^{(0)}(r) \right]. \end{aligned} \quad (41)$$

In the small  $\frac{\omega}{r^z}$  limit, the asymptotical form of the inhomogeneous solution is

$$X_{\omega}^{(\phi)}(r) = \frac{r^{-2-3z}}{2z(2+3z)} \int d\omega' \omega' (\omega - \omega') \left( \frac{\omega'}{z} \right)^{\frac{2+2z}{z}} \varphi(\omega') \quad (42)$$

with  $\varphi(\omega)$  given by (37). Thus, the corrections to the Nambu-Goto action due to the gravitation waves in (38) have the explicit  $\phi$  dependence and the contributions from  $X_{\omega}^{(\phi)}(r)$ . Since  $\phi(\omega, r_b) \gg X_{\omega}^{(\phi)}(r_b)/X_{\omega}^{(0)}(r_b)$  for large  $r_b$ , the contributions from  $X_{\omega}^{(\phi)}$  to the above perturbed action (38) can be ignored if we keep the leading-order terms in small  $\phi$ . The on-shell perturbed action  $S_{NG\phi}^{\text{on-shell}}$  is then expressed as

$$\begin{aligned} S_{NG\phi}^{\text{on-shell}} &= -\frac{r_b^{3+z}}{4\pi\alpha'} \int dt \phi(t, r_b) (Q^+(t, r_b) \partial_r Q^+(t, r_b) - Q^-(t, r_b) \partial_r Q^-(t, r_b)) \\ &= -\frac{r_b^{3+z}}{4\pi\alpha'} \int \frac{d\omega}{2\pi} \int \frac{d\omega'}{2\pi} \phi(\omega + \omega', r_b) (Q_{-\omega}^+(r_b) \partial_r Q_{-\omega'}^+(r_b) - Q_{-\omega}^-(r_b) \partial_r Q_{-\omega'}^-(r_b)) . \end{aligned} \quad (43)$$

The holographic perturbed influence functional will be found when we substitute into the above expression the zero- $T$  limit of (29) with  $\mathcal{X}_{\omega}(r)$  given by (31). The corrections to the nonequilibrium Green's functions can be identified in this perturbed holographic influence functional. It will be shown later that the possible holographic description of squeezed vacuum states is given by the gravitation wave perturbations. Thus, these nonequilibrium Green's functions obtained from the perturbed influence functional will be compared with the Green's functions of general multi-mode squeezed vacuum states of the boundary fields. In so doing, the function  $\varphi$  in (37), determined by the boundary condition of gravitation waves, will be related to the squeezing parameters of the squeezed vacuum states.

### III. VELOCITY FLUCTUATIONS OF A PARTICLE

In principle the effects from vacuum fluctuations of an environment field on a particle can be revealed in the particle's velocity dispersion. The environment field not only modifies the evolution of the particle's mean trajectory but also introduces additional stochastic motion [10]. These two effects are encoded in the associated Langevin equation. To derive this equation from the influence functional (7), we find it more convenient to change the  $q^+$ ,  $q^-$  coordinates to the average and relative coordinates:

$$q = (q^+ + q^-)/2, \quad q_{\Delta} = q^+ - q^- . \quad (44)$$

As will be seen later, the influence of the environment field can give the mass of the particle, and damp its motion. As such, here we consider that all terms associated with the particle are dynamically generated from the contributions of the environmental quantum fields. Thus, the coarse-grained effective action can be defined from (6) with the influence functional (7) only, and thus there is no need to introduce the Lagrangian  $L_q[q]$  of the particle:

$$\begin{aligned} S_{\text{CG}}[q^\pm = q \pm q_\Delta/2] &= -i \ln \mathcal{F}[q^+, q^-] \\ &= \int dt q_\Delta(t) \left[ - \int dt' G_R(t, t') q(t') \right] + \frac{i}{2} \int dt \int dt' q_\Delta(t) G_H(t, t') q_\Delta(t'). \end{aligned} \quad (45)$$

We then further introduce an auxiliary variable  $\eta(t)$ , the noise force, with a Gaussian distribution function:

$$P[\eta(t)] = \exp \left\{ -\frac{1}{2} \int dt \int dt' \eta(t) G_H^{-1}(t, t') \eta(t') \right\}. \quad (46)$$

In terms of the noise force  $\eta(t)$ ,  $S_{\text{CG}}$  can be rewritten as an ensemble average over  $\eta(t)$ ,

$$\exp i S_{\text{CG}} = \int \mathcal{D}\eta P[\eta(t)] \exp i S_\eta[q, q_\Delta; \eta], \quad (47)$$

where the stochastic coarse-grained effective action  $S_\eta$  is given by

$$S_\eta[q, q_\Delta; \eta] = \int dt q_\Delta(t) \left[ - \int dt' G_R(t, t') q(t') + \eta(t) \right]. \quad (48)$$

Varying the action  $S_\eta$  with respect to  $q_\Delta$  and setting  $q_\Delta = 0$  will give the Langevin equation of  $(q(t) = X(t))$ :

$$\int dt' G_R(t, t') X(t') = \eta(t). \quad (49)$$

The noise force correlation function can be obtained from (46) as

$$\langle \eta(t) \eta(t') \rangle = G_H(t, t'). \quad (50)$$

Evidently, the retarded Green's function will modify the trajectory, given by noise or external forces, and the correlated noise forces will render the trajectory fluctuating. Since the low frequency expansion of the retarded Green's function at zero- $T$  in (32) is [33, 40]

$$G_R^{(0)}(\omega) = m(z)(i\omega)^2 + \mu(\omega, z), \quad (51)$$

for a general  $z$  the induced mass  $m$  and the  $\mu$  term are given by

$$m(z) = \frac{1}{\pi \alpha' (2 - z) r_b^{z-2}}, \quad \mu(\omega, z) = \gamma(z) (-i\omega)^{1+\frac{2}{z}} + \mathcal{O}(\omega^4) \quad (52)$$

with the damping coefficient

$$\gamma(z) = \frac{1}{\pi\alpha'(2z)^{2/z}} \frac{\Gamma(\frac{1}{2} - \frac{1}{z})}{\Gamma(\frac{1}{2} + \frac{1}{z})}. \quad (53)$$

Although both  $m$  and  $\gamma$  change their signs at  $z = 2$ , the ratio  $\gamma/m$  remains positive and varies continuously at  $z = 2$ . When  $z > 2$ , the minus signs of the mass term and the term associated with damping can be simultaneously removed in the dynamical equation (49) by changing the sign of the noise forces, i.e.  $\eta \rightarrow -\eta$ , thus leading to a sensible equation of motion. In fact, the noise forces are introduced as auxiliary variables and their effects on the dynamics of the particle are formulated only in the form of the correlation functions. Then, the magnitude of  $m$  can be identified as a dynamical mass of the particle, which can be large, about the order  $\lambda \propto 1/\alpha'$ .  $\lambda$  corresponds to the coupling constant in quantum field theory via AdS/CFT correspondence.

Before proceeding further, it is of interest to see how the solution to the Langevin equation is connected with the fluctuation-dissipation relation in [33]. In the case of the vacuum state of an environment field, the Fourier transform of the Langevin equation (49) is expressed as

$$X(\omega) = \eta(\omega)/G_R^{(0)}(\omega). \quad (54)$$

Then, the fluctuations in the position of a particle induced by noise forces are obtained as

$$\langle X(\omega)X(-\omega) \rangle = \frac{\langle \eta(\omega)\eta(-\omega) \rangle}{G_R^{(0)}(\omega)G_R^{(0)*}(\omega)} = \frac{G_H^{(0)}(\omega)}{G_R^{(0)}(\omega)G_R^{(0)*}(\omega)} = -\text{Im}\chi(\omega), \quad (55)$$

where  $\chi^{-1}(\omega) = -G_R^{(0)}(\omega)$  and the fluctuation-dissipation relation (14) in the  $T \rightarrow 0$  limit is applied. The above expression is a key result in [33]. Here we recover it by solving the Langevin equation, which is derived from the obtained influence functional.

To proceed for the squeezed vacuum states, we consider a quantized string, as in [33], with its mode expansion as follows:

$$X(t) = X(t, r_b) = \sqrt{\frac{\pi\alpha'}{z}} \int_0^\infty d\omega U_\omega \left[ a_\omega e^{-i\omega t} + a_\omega^\dagger e^{+i\omega t} \right], \quad (56)$$

where  $a_\omega$  and  $a_\omega^\dagger$  are the annihilation and creation operators, and they obey canonical commutation relations. In the background of Lifshitz black hole, the string perturbations are in thermal states where  $\langle a_\omega^\dagger a_\omega \rangle = (e^{\frac{\omega}{T}} - 1)^{-1}$  with black hole temperature  $T$ . The mode functions in zero- $T$  limit becomes

$$U_\omega = \frac{1}{\sqrt{1 + C_\omega^2}} \frac{1}{r_b^{1+\frac{z}{2}}} \left[ J_{\frac{1}{2}+\frac{1}{z}}\left(\frac{\omega}{zr_b^z}\right) + C_\omega Y_{\frac{1}{2}+\frac{1}{z}}\left(\frac{\omega}{zr_b^z}\right) \right], \quad (57)$$

in which

$$C_\omega = -\frac{J_{-\frac{1}{2}+\frac{1}{z}}\left(\frac{\omega}{zr_b^z}\right)}{Y_{-\frac{1}{2}+\frac{1}{z}}\left(\frac{\omega}{zr_b^z}\right)}. \quad (58)$$

Thus it is quite reasonable to assume that the squeezed vacuum states can be constructed from the Bogoliubov transformations of the creation and annihilation operators of the normal vacuum state. Here we consider the two-mode squeezed states where the squeezed operator is defined as

$$|\xi_{\omega\omega'}\rangle = S(\xi_{\omega\omega'}) |0\rangle, \quad S(\xi_{\omega\omega'}) = \exp\left[\frac{1}{2}(\xi_{\omega\omega'}^* a_\omega a_{\omega'} - \xi_{\omega\omega'} a_\omega^\dagger a_{\omega'}^\dagger)\right] \quad (59)$$

with the squeezing parameter  $\xi_{\omega\omega'} = r_{\omega\omega'} e^{i\theta_{\omega\omega'}}$ . With the help of the Baker-Campbell-Hausdorff formula, we readily find the Bogoliubov transformations of the creation and annihilation operators due to the squeeze operator  $\mathcal{S}(\xi_{\omega\omega'})$ ,

$$\begin{aligned} \mathcal{S}^\dagger(\xi_{\omega\omega'}) a_\omega \mathcal{S}(\xi_{\omega\omega'}) &= \mu_{\omega\omega'} a_\omega - \nu_{\omega\omega'} a_{\omega'}^\dagger, & \text{and} & & \mathcal{S}^\dagger(\xi_{\omega\omega'}) a_{\omega'}^\dagger \mathcal{S}(\xi_{\omega\omega'}) &= \mu_{\omega\omega'} a_{\omega'}^\dagger - \nu_{\omega\omega'}^* a_\omega, \\ \mathcal{S}^\dagger(\xi_{\omega\omega'}) a_{\omega'} \mathcal{S}(\xi_{\omega\omega'}) &= \mu_{\omega\omega'} a_{\omega'} - \nu_{\omega\omega'} a_\omega^\dagger, & \text{and} & & \mathcal{S}^\dagger(\xi_{\omega\omega'}) a_\omega^\dagger \mathcal{S}(\xi_{\omega\omega'}) &= \mu_{\omega\omega'} a_\omega^\dagger - \nu_{\omega\omega'}^* a_{\omega'}, \end{aligned} \quad (60)$$

and we have

$$\langle \xi_{\omega\omega'} | a_\omega | \xi_{\omega\omega'} \rangle = 0, \quad \langle \xi_{\omega\omega'} | a_\omega a_{\omega'} | \xi_{\omega\omega'} \rangle = -\mu_{\omega\omega'} \nu_{\omega\omega'}, \quad \langle \xi_{\omega\omega'} | a_\omega^\dagger a_{\omega'} | \xi_{\omega\omega'} \rangle = \eta_{\omega\omega'}^2 2\pi \delta(\omega - \omega'), \quad (61)$$

where  $\mu_{\omega\omega'} = \cosh r_{\omega\omega'}$ ,  $\nu_{\omega\omega'} = e^{i\theta_{\omega\omega'}} \sinh r_{\omega\omega'}$  and  $\eta_{\omega\omega'} = |\nu_{\omega\omega'}|$ . Notice that the retarded Green's function defined in (9) remains the same in the two-mode squeezed vacuum state because the involved Bogoliubov transformations are the canonical ones so they preserve the commutation relations between the creation and annihilation operators. Then, the position correlator  $\langle X(t) X(t') \rangle$  in the squeezed vacuum states can be calculated straightforwardly. Using the Langevin equation in (49), which is also valid for a particle influenced by the environmental quantum fields in general quantum states, we can find the corresponding Hadamard function of the boundary fields in the squeezed vacuum states  $G_H^{(s)}(t, t')$

$$\begin{aligned} G_H^{(s)}(t, t') &= \frac{\pi\alpha'}{2} \int_0^\infty \frac{d\omega}{2\pi} \int_0^\infty \frac{d\omega'}{2\pi} W(\omega) W(\omega') U_\omega U_{\omega'} G_R^{(0)}(\omega) G_R^{(0)*}(\omega') \\ &\quad \left[ -\mu_{\omega\omega'} \nu_{\omega\omega'} \frac{G_R^{(0)}(\omega')}{G_R^{(0)*}(\omega')} e^{-i\omega t - i\omega' t'} + 2\pi \delta(\omega - \omega') \left( \eta_{\omega\omega'}^2 + \frac{1}{2} \right) \left( e^{-i\omega t + i\omega' t'} \right) \right] + \text{h.c.}, \end{aligned} \quad (62)$$

where  $G_R^{(0)}$  is the Fourier transform of the retarded Green's function in the normal vacuum state obtained by (32). In the above expression, we have introduced the simplest window function  $W(\omega)$  given by the unit-step functions

$$W(\omega) = 1, \quad \text{if } \omega_0 - \Delta \leq \omega \leq \omega_0 + \Delta. \quad (63)$$

Thus only modes within the frequency band  $\omega_0 - \Delta \leq \omega \leq \omega_0 + \Delta$  are excited to the squeezed vacuum. The other modes remain in normal vacuum. We can choose a more smooth window function as long as it falls off to zero sufficiently fast outside the frequency band of interest. Apparently, in (62) there are two distinct contributions to the Green's function. The second terms of (62) are the stationary component. However, there exists a nonstationary component, and a time-dependent external field is required to squeeze the vacuum. From the perturbed holographic influence functional derived in (43), the corrections to the Hadamard function of boundary fields in vacuum (33), denoted by  $G_H^{(\phi)}(t, t')$ , can be obtained as

$$G_H^{(\phi)}(t, t') = \int_0^\infty \frac{d\omega}{2\pi} \int_0^\infty \frac{d\omega'}{2\pi} 2 G_H^{(0)}(\omega) \left\{ \phi(\omega + \omega', r_b) e^{-i\omega t - i\omega' t'} + \phi(\omega - \omega', r_b) e^{-i\omega t + i\omega' t'} + \text{h.c.} \right\}. \quad (64)$$

In the limits of small squeezing parameters and the narrow bandwidth ( $\Delta/\omega_0 < 1$ ) in (63), when  $\omega$  and  $\omega'$  lie within the frequency band, we have  $\omega \approx \omega'$ . Compared (64) with (62), the field  $\varphi(2\omega)$  obtained from  $\phi(\omega, r_b)$  in (37) can be related to the squeezing parameters up to a constant phase by

$$r_b^{-2-z} \varphi(\omega + \omega' \approx 2\omega) = -r_{\omega\omega} \Gamma\left(\frac{3}{2} + \frac{1}{z}\right) \left(\frac{\omega}{z}\right)^{-\frac{2+z}{2z}}, \quad (65)$$

where the mode functions (57) can be expressed in terms of the retarded Green's function and Hadamard function as

$$\frac{\pi^2 \alpha'}{2} U_\omega^2 = \frac{G_H^{(0)}(\omega)}{G_R^{(0)}(\omega) G_R^{(0)*}(\omega)}. \quad (66)$$

The large  $r_b$  limit is taken in (65). For a large but finite  $r_b$ , the squeezing parameters are typically small.

In the following we will consider the subvacuum effects for small squeezing parameters so as to be consistent with the approximations we adopted here. The above identification provides a possible scheme for the dual gravity theory to generate metric perturbations that

may correspond to the squeezed vacuum states of the boundary fields, as suggested in [56]. The physical picture is that the gravitation waves are generated somewhere at small  $r$ , and then propagate toward the boundary at larger  $r$ . On their way to reach the boundary, the waves induce fluctuations to the string dynamics, and thus excite the quantum state of a string in such a way that their effects on the end point can be interpreted as those due to the squeezed vacuum fluctuations of the boundary field. However the faithful identification of the dual description needs the order by order comparison of the correlators, obtained from bulk theory and boundary field theory, in terms of small squeezing parameters. This work is underway.

If we choose the frequency-independent squeezing parameters  $\xi_{\omega,\omega'} = 2\pi\xi\delta(\omega - \omega')$  within the frequency band, the difference between the velocity dispersion due to squeezed vacuum and normal vacuum, defined by  $\delta\langle(\Delta V)^2\rangle_\xi \equiv \langle(V(t) - V(0))^2\rangle_\xi - \langle(V(t) - V(0))^2\rangle_0$ , can be derived directly from the mode expansion (56),

$$\begin{aligned}\delta\langle(\Delta V)^2\rangle_\xi &= \int_0^\infty \frac{d\omega}{2\pi} W(\omega) \frac{2G_H^{(0)}(\omega)}{G_R^{(0)}(\omega)G_R^{(0)*}(\omega)} \omega^2 \left[ +\mu_\omega\nu_\omega (e^{-i\omega t} - 1)^2 \right. \\ &\quad \left. +\mu_\omega\nu_\omega^* (e^{+i\omega t} - 1)^2 + 2\eta_\omega^2 (e^{-i\omega t} - 1)(e^{+i\omega t} - 1) \right] \\ &= \int_{\omega_0-\Delta}^{\omega_0+\Delta} \frac{d\omega}{2\pi} \frac{G_H^{(0)}(\omega)}{G_R^{(0)}(\omega)G_R^{(0)*}(\omega)} \omega^2 16 \left[ -\mu\eta \cos[\omega t - \theta] + \eta^2 \right] \sin^2 \frac{\omega t}{2}, \quad (67)\end{aligned}$$

where the mode functions can be written in terms of the retarded Green's function and Hadamard function (66), and the window function (63) is introduced. Thus, the above expression of the shifted velocity has taken account of not only the stochastic effects of the environmental quantum fields through the Hadamard function, but also the dissipative effects of the retarded Green's function. The saturated value of the shifted velocity dispersion can be found from the late-time behavior of (67) in the limit  $(\omega_0 \pm \Delta)t \gg 1$ . In this case the main contributions to the  $\omega$ -integration come from the regions of small  $\omega$ . The small  $\omega$  expansion of  $G_H^{(0)}(\omega)/(G_R^{(0)}(\omega)G_R^{(0)*}(\omega))$  takes different forms for  $1 < z < 2$  and  $z > 2$ , and



they are respectively given by

$$\begin{aligned} \frac{G_H^{(0)}(\omega)}{G_R^{(0)}(\omega)G_R^{(0)*}(\omega)} &= \frac{4\alpha' z r_b^{z-2}}{\omega^2} \left[ J_{\frac{1}{z}-\frac{1}{2}}^2\left(\frac{\omega}{z r_b^z}\right) + Y_{\frac{1}{z}-\frac{1}{2}}^2\left(\frac{\omega}{z r_b^z}\right) \right]^{-1} \\ &\simeq \begin{cases} \frac{2}{\alpha'} \mathcal{N}_{1 < z < 2} m^{-2} \omega^{-3+2/z}; & \mathcal{N}_{1 < z < 2} = \frac{1}{\Gamma^2(\frac{1}{z} + \frac{1}{2})}, & 1 < z < 2, \\ 2\pi^2 \alpha' \mathcal{N}_{z > 2} \omega^{-1-2/z}; & \mathcal{N}_{z > 2} = \frac{1}{\Gamma^2(-\frac{1}{z} + \frac{1}{2})}, & z > 2. \end{cases} \end{aligned} \quad (68)$$

There is a dramatic change at  $z = 2$ . Notice that different  $\omega$  dependence in these two regimes of  $z$  is mainly attributed to the fact that the low frequency behavior of the retarded Green's function is dominated by the inertial mass term when  $1 < z < 2$  and by the  $\gamma$  term when  $z > 2$ .

The change in the velocity dispersion at late time is given by

$$\delta\langle(\Delta V)^2\rangle_\xi \simeq \begin{cases} \frac{4\mathcal{N}_{1 < z < 2}}{\pi\alpha'} g_+(r, \theta) \frac{[(\omega_0 + \Delta)^{2/z} - (\omega_0 - \Delta)^{2/z}]}{m^2} + \mathcal{O}((\omega_0 \pm \Delta)^{-1+2/z}/m^2 t), & 1 < z < 2; \\ \frac{4\pi\alpha'\mathcal{N}_{z > 2}}{(z-1)} g_+(r, \theta) \left[ (\omega_0 + \Delta)^{2-2/z} - (\omega_0 - \Delta)^{2-2/z} \right] + \mathcal{O}((\omega_0 \pm \Delta)^{1-2/z}/t), & z > 2, \end{cases} \quad (69)$$

where the function  $g_\pm$  of squeezing parameters is defined as

$$g_\pm(\eta, \theta) = 2\eta^2 \pm \eta\mu \cos(\theta). \quad (70)$$

Thus, the evolution of the shifted velocity dispersion of the particle due to the squeezed vacuum of environment fields will reach a saturated value at late times, following a power-law like  $t^{-1}$ .

Notice that for some particular choices of squeezing parameters, the function  $g(\eta, \theta)$  can be negative, leading to the so-called subvacuum phenomenon. The most negative value can be found as

$$\eta^2 - \frac{1}{2}\eta\mu \geq -\frac{2-\sqrt{3}}{4} > -\frac{1}{2}. \quad (71)$$

Therefore, the subvacuum effect has a lower bound given by the inequality above. This is expected because the sum of the velocity dispersion arising from the normal vacuum and the shifted value due to the squeezed vacuum must be positive. Thus, the shifted value has to be greater than the minus of the result from the pure vacuum, which in turn constrains how negative the shifted value can reach.

In the case of  $1 < z < 2$ , the subvacuum phenomenon shown in the velocity dispersion is found to have the  $1/m^2$  dependence, and is consistent with the findings in weakly coupled fields [11] for the case of  $z = 1$ . Although this subvacuum effect is enhanced by a strongly coupling constant  $\lambda \propto 1/\alpha'$ , the heavy mass dependence  $m^2 \propto \lambda^2$  will suppress this effect. As to the  $z > 2$  case, since the dominant term of the retarded Green's function comes from the  $\gamma$  term in (51) and (53), the coupling constant dependence in the  $\gamma$  term leads to the  $1/\lambda$  dependence of the subvacuum effect, which is also relatively weak for a large  $\lambda$ . To make an estimate, the typical length scale  $L$ , which is associated with the breakdown of Lorentz invariance in quantum critical theory, is introduced [52]. If the scale  $1/L$  is the largest momentum scale in the system, the large mass  $m$  can be parameterized as  $m = \lambda/L$ . Then, we find that

$$|\delta\langle(\Delta V)^2\rangle_\xi| \simeq \begin{cases} \frac{1}{\lambda} \left(\frac{L}{\lambda_0}\right)^{2/z} \left(\frac{\Delta}{\omega_0}\right), & 1 < z < 2; \\ \frac{1}{\lambda} \left(\frac{L}{\lambda_0}\right)^{2-2/z} \left(\frac{\Delta}{\omega_0}\right), & z > 2, \end{cases} \quad (72)$$

where  $\lambda_0$  is a typical wavelength of squeezed vacuum modes, and  $\lambda_0 > L$  in general. The result of the shifted velocity dispersion is suppressed by a large  $\lambda$ , but in principle observable.

#### IV. DECOHERENCE AND RECOHERENCE

The above influence functional can also be applied to study the nature of quantum coherence of a particle when it interacts with the environment. We consider the initial state  $|\Psi(t_i)\rangle$  of the particle to be a coherent superposition of two localized states. Additionally, both states can be arranged to have the same spatial point at the moment  $t_i$ ,

$$|\Psi(t_i)\rangle = |\psi_1(t_i)\rangle + |\psi_2(t_i)\rangle, \quad (73)$$

then the initial density matrix of the state can be written as

$$\rho_q(t_i) = |\Psi(t_i)\rangle\langle\Psi(t_i)| = \rho_{11}(t_i) + \rho_{22}(t_i) + \rho_{21}(t_i) + \rho_{12}(t_i), \quad (74)$$

where  $\rho_{mn}(t_i) = |\psi_m(t_i)\rangle\langle\psi_n(t_i)|$ . We assume that these two localized states  $|\psi_m\rangle$  move along their respective paths  $C_m$  such that they leave from the same spatial point and recombine at the location  $q_f$  at later time  $t_f$ . The interference pattern of the superposed state

at  $t_f$  is given by the off-diagonal elements of the reduced density matrix. Here we assume that the de Broglie wavelength of the localized states is much shorter than the length scale of interest so that the width of the wavefunctions and their subsequent spreading can be legitimately neglected [12]. Thus, the leading effect of the reduced density matrix can be obtained by evaluating the propagating function (6) along a mean trajectory of the localized states dictated by an external force. Then  $\rho_r(q_f, q_f, t_f)$  now becomes

$$\rho_r(q_f, q_f, t_f) = |\psi_1(q_f, t_f)|^2 + |\psi_2(q_f, t_f)|^2 + 2e^{\mathcal{W}[\bar{q}^+, \bar{q}^-]} \text{Re} \left\{ e^{i\Phi[\bar{q}^+, \bar{q}^-]} \psi_1(q_f, t_f) \psi_2^*(q_f, t_f) \right\}, \quad (75)$$

where the  $\mathcal{W}$  and  $\Phi$  functionals are the phase and modulus of the influence functional defined by:

$$\mathcal{F}[q^+, q^-] = \exp \left\{ \mathcal{W}[q^+, q^-] + i\Phi[q^+, q^-] \right\}, \quad (76)$$

and they are

$$\Phi[\bar{q}^+, \bar{q}^-] = -\frac{1}{2} \int dt \int dt' \left[ \bar{q}^+(t) - \bar{q}^-(t') \right] G_R(t, t') \left[ \bar{q}^+(t) + \bar{q}^-(t') \right], \quad (77)$$

$$\mathcal{W}[\bar{q}^+, \bar{q}^-] = -\frac{1}{2} \int dt \int dt' \left[ \bar{q}^+(t) - \bar{q}^-(t') \right] G_H(t, t') \left[ \bar{q}^+(t) - \bar{q}^-(t') \right], \quad (78)$$

evaluated along the classical trajectories,  $C_1 = \bar{q}^+$  and  $C_2 = \bar{q}^-$ . The modulus of the influence functional  $\mathcal{W}$  reveals decoherence between the coherently superposed states, and the phase functional  $\Phi$  results in an overall phase shift for the interference pattern. Both effects to the particle states arise from the interaction with quantum fields. The retarded Green's function and Hadamard function constructed out of quantum critical fields have been obtained by the holographic method.

In [13, 48], we studied the decoherence effect to a quantum particle from the vacuum state of electromagnetic fields and quantum critical fields respectively. However from what we have learned above, the retarded Green's function of strongly coupled quantum critical fields plays an essential role in determining the trajectory, otherwise, driven by an external or noise force. In addition, the terms that dominate in the regime  $1 < z < 2$  and  $z > 2$  give rise to rather different relaxation behaviors of the particle. Accordingly, we will reexamine this effect by taking account of the retarded Green's function properly into the equation of motion. So, for a prescribed force whose Fourier transform is defined by  $F_{ex}(\omega) \equiv m\omega^2\zeta(\omega)$ , the trajectory follows the solution of (54) is,

$$q_\zeta(\omega) = m\omega^2\zeta(\omega)/G_R(\omega), \quad (79)$$

where the form of  $\zeta$  will be specified later. We consider the mean trajectories of two localized states specified by  $\bar{q}^\pm = \pm q_\zeta$  in the interference experiment and then for the time-translation invariant Green's function, the decoherence functional is given by

$$\mathcal{W} = -2 \int \frac{d\omega}{2\pi} \frac{m \omega^2 \zeta(\omega)}{G_R(\omega)} G_H(\omega) \frac{m \omega^2 \zeta(-\omega)}{G_R^*(\omega)}. \quad (80)$$

The function  $\zeta(t)$  is required to be sufficiently smooth and is chosen to take the form [12],

$$\zeta(t) = \frac{\ell_0}{\tau_0^4} (t^2 - \tau_0^2)^2, \quad (81)$$

where  $2\ell_0$  characterizes a length scale for path separation and  $2\tau_0$  is the effective flight time for  $-\tau_0 < t < \tau_0$  as in [13]. These two prescribed classical trajectories are symmetric with respect to the initial position so that  $q^+(t) + q^-(t) = 0$  and thus we will not see any phase shift,  $\Phi = 0$  from (77). We now focus on the decoherence functional  $\mathcal{W}$  contributed by the quantum critical fields. For the environment in its normal vacuum state, the retarded Green's function  $G_R^{(0)}$  and Hadamard function  $G_H^{(0)}$  are given in (32) and (33) respectively, and then the decoherence functional is found to be

$$\mathcal{W}_0 \simeq \begin{cases} -\frac{2\mathcal{N}_{1<z<2}^{\mathcal{W}_0}}{\pi\alpha'} \frac{\ell_0^2}{\tau_0^{2/z}} + \mathcal{O}(1/r_b^2\tau_0^2), & 1 < z < 2; \\ -2\pi\alpha' \mathcal{N}_{z>2}^{\mathcal{W}_0} \frac{m^2\ell_0^2}{\tau_0^{2-2/z}} + \mathcal{O}(1/r_b^2\tau_0^2), & z > 2, \end{cases} \quad (82)$$

where

$$\begin{aligned} \mathcal{N}_{1<z<2}^{\mathcal{W}_0} &= 4\mathcal{N}_{1<z<2} \frac{2^{12-\frac{2}{z}}}{z^2(4z-1)} (5z-2)(z-1) \cos\left(\frac{\pi}{z}\right) \Gamma\left(-6+\frac{2}{z}\right), \\ \mathcal{N}_{z>2}^{\mathcal{W}_0} &= 4\mathcal{N}_{z>2} \frac{2^{10+\frac{2}{z}}}{z^2(3z+1)} (3z+2)(z-1) \cos\left(\frac{\pi}{z}\right) \Gamma\left(-4-\frac{2}{z}\right). \end{aligned}$$

The negative value of the  $\mathcal{W}$  function indicates the decoherence effect on the wavefunction of the particle. Thus, in the case  $1 < z < 2$ , for a fixed travel time  $\tau_0$ , the magnitude of  $|\mathcal{W}_0|$  increases as  $z$  increases, and a large coupling constant  $\lambda \propto 1/\pi\alpha'$  renders significant decoherence. As for  $z > 2$ , the combined results from the respective  $1/\lambda$  and  $m^2 \propto \lambda^2$  dependence also cause large decoherence. In this range of  $z$ , the value of  $|\mathcal{W}_0|$  decreases as  $z$  increases instead. The most significant decoherence effect occurs at  $z = 2$ . For  $z = 1$ , the result is consistent with the weakly coupled relativistic field case apart from its large coupling constant dependence, giving large decoherence.

Now we study the  $\mathcal{W}$  function associated with the squeezed vacuum state. The above expression (80) needs to be modified to account for the non-stationary component of the Green's function. The associated decoherence functional is defined by  $\mathcal{W}_\xi = \mathcal{W}_0 + \delta\mathcal{W}_\xi$  where  $\delta\mathcal{W}_\xi$  is the modification from the result due to the normal vacuum of the field, and is given by

$$\delta\langle\mathcal{W}\rangle_\xi = -16 \int_{\omega_0-\Delta}^{\omega_0+\Delta} \frac{d\omega}{2\pi} \frac{G_H^{(0)}(\omega)}{G_R^{(0)}(\omega)G_R^{(0)*}(\omega)} m^2 \omega^4 \zeta(\omega) \zeta(-\omega) [\mu \eta \cos[\omega t - \theta] + \eta^2]. \quad (83)$$

The straightforward calculations show at large time scales  $\tau_0 \gg 1/\omega_0$ ,

$$\delta\mathcal{W}_\xi \simeq \begin{cases} -\frac{2\mathcal{N}_{1 < z < 2}}{\pi\alpha'} g_-(r, \theta) \frac{\ell_0^2}{\tau_0^{2/z}} \frac{1024}{(\omega_0\tau_0)^{4-2/z}} \frac{\Delta}{\omega_0} \left[ \sin(\omega_0\tau_0) + \mathcal{O}(1/(\omega_0\tau_0)) \right], & 1 < z < 2; \\ -2\pi \alpha' \mathcal{N}_{z > 2} g_-(r, \theta) m^2 \frac{\ell_0^2}{\tau_0^{2-2/z}} \frac{1024}{(\omega_0\tau_0)^{2+2/z}} \frac{\Delta}{\omega_0} \left[ \sin(\omega_0\tau_0) + \mathcal{O}(1/(\omega_0\tau_0)) \right], & z > 2, \end{cases} \quad (84)$$

where we have assumed the narrow bandwidth, namely  $\omega_0 > \Delta$ . Thus, by choosing some particular values of squeezing parameters,  $g_-(r, \theta)$  in (70) can be negative with the most negative value  $-(2 - \sqrt{3})/2$ . Thus, an increase of contrast, as compare to that from the normal vacuum, is seen for  $\delta\mathcal{W}_\xi > 0$  due to  $g_-(r, \theta) < 0$  for certain values of the squeezing parameters. This is the phenomenon of recoherence, which is another subvacuum effect. For  $1 < z < 2$ , a strong coupling  $\lambda \propto 1/\alpha'$  may render an enhancement on this recoherence effect. For  $z > 2$ , this subvacuum effect is also enhanced due to large  $m^2/\lambda \propto \lambda$  dependence. To have a rough estimate, we consider the typical frequency of the squeezed vacuum modes such that  $(1/\omega_0) \equiv \lambda_0 \sim \ell_0$  [12]. When  $g_-(r, \theta) < 0$  in the expression (84) for appropriate squeezing parameters, an order of the magnitude estimation of the recoherence phenomenon is found to be

$$\delta\mathcal{W}_\xi \simeq \begin{cases} \lambda \left(\frac{\ell_0}{\tau_0}\right)^4 \left(\frac{\ell_0}{L}\right)^{2-2/z} \left(\frac{\Delta}{\omega_0}\right), & 1 < z < 2; \\ \lambda \left(\frac{\ell_0}{\tau_0}\right)^4 \left(\frac{\ell_0}{L}\right)^{2/z} \left(\frac{\Delta}{\omega_0}\right), & z > 2, \end{cases} \quad (85)$$

where we have again parameterized  $m \simeq \lambda/L$ , and  $L$ , the Lorentz symmetry breaking scale in quantum critical field theory, is introduced. In the  $z = 1$  case, the  $(\ell_0/\tau_0)^4$  dependence is consistent with the findings from an environment of a weakly coupled relativistic field [12]. For a general  $z$ , this phenomenon will also depend on  $\ell_0/L$ . When  $\ell_0 > L$ , the maximum

recoherence effect is found at  $z = 2$  whereas the minimum effect happens at  $z = 1$  and  $z \rightarrow \infty$ . The presence of vacuum fluctuations of strongly coupled quantum critical fields in squeezed state gives rise to potentially large recoherence effect on the wavefunction of a particle. Thus the large coupling constant may offer the possibility to observe these subvacuum phenomena.

## V. SUMMARY AND OUTLOOK

Using the holographic influence functional approach, the effects of the strongly coupled quantum critical fields on the dynamics of a particle are studied. The dual description is a string moving in 4+1-dimensional Lifshitz geometry. We first study the influence on the velocity dispersion of a particle from the squeezed vacuum states of the quantum critical fields. We find that the evolution of the velocity dispersion will reach a constant at late times where its relaxation dynamics follows a power law in time as  $t^{-1}$ . With particular choices of squeezing parameters, the saturated value is found to be smaller than the value given by the normal vacuum background. This leads to a subvacuum effect. The reduction in the velocity dispersion changes dramatically as the dynamical exponent passes through  $z = 2$ . This subvacuum effect, which is found proportional to  $1/\lambda \propto \alpha'$ , is suppressed by a large value  $\lambda$  in quantum field theory, but is in principle observable. We then study the decoherence dynamics of a quantum particle induced also by the squeezed vacuum fluctuations. We find that coherence loss can be shown less severe in certain squeezed vacuums than in normal vacuum. This recovery of coherence is understood as recoherence, another manifestation of the subvacuum phenomena. We make some estimates of the degree of recoherence, which is enhanced by a large coupling constant  $\lambda$ , and thus can potentially be measurable. We also show that there exists a bound to constrain the above-mentioned subvacuum phenomena.

Finally, we would like to point out some of our future works. In view of a close relation between the holographic approach and the field-theoretical study, an important next step is to study the issue of negative energy density and find the associated quantum inequality, which constrains the magnitude and duration of the negative energy density in the strongly coupled field, using the method of the holography principle. This might give a profound implication to the existence of exotic spacetimes sourced by negative energy.

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